Response Surface Models for the Uncertainty Quantification of Eccentric Permanent Magnet Synchronous Machines

Zeger Bontinck[1,2], Herbert De Gersem[2], and Sebastian Schöps[1,2],

[1]Graduate School of Computational Engineering, Technische Universität Darmstadt, Darmstadt, Germany
[2]Institut Theorie Elektromagnetischer Felder, Technische Universität Darmstadt, Darmstadt, Germany

This work deals with the modelling and simulation of the effect of rotor eccentricity in permanent magnet synchronous machines. Static eccentricity is analyzed in a 2D setting. The 3D effect of an inclined rotor shaft is accounted for considering 2D slices and interpolating on a grid constructed from finite element simulations (response surface model). Common tools of uncertainty quantification, i.e. generalized polynomial chaos and Monte Carlo, are used to study the effect on the electromotive force. The focus of the abstract is the construction of the response surface models used.

***Index Terms*—Finite element analysis, Monte carlo methods, permanent magnet machines, response surface methodology**

I. Introduction

THE MACHINE used in this work is a 3-phase 6-pole permanent magnet synchronous machine (PMSM). The uncertainties handled are linked to the position of the rotor within the stator, known as eccentricity. One of the undesired effects of eccentricity is for example the production of noise [1]. To determine the influence of eccentricity the electromotive force (EMF) in the stator windings is calculated by using the standard techniques from uncertainty quantification (UQ), i.e. generalized Polynomial Chaos (gPC) and the Monte-Carlo (MC) approach. In this work different ways to construct response surface models (RSMs) are discussed.

II. Methodology

A. Eccentricity models

1) Static eccentricity

The first model deals with uncertain static eccentricity (SE). A coordinate system is assigned to the nominal position of the rotor, i.e. in the centre of the stator. The position of the rotors centre in the static eccentric case is described in polar coordinates, i.e. R depicts the magnitude and θ the angle of displacement. $R(\omega)$ and $\theta(\omega)$ are independent random variables on the probability space (Θ, Σ, P). R and θ are modelled as Gaussian and uniformly distributed, respectively:

$$R \sim \mathcal{N}(0, \sigma^2) \quad \text{and} \quad \theta \sim \mathcal{U}(0, \frac{\pi}{3}), \tag{1}$$

where σ is the standard deviation such that 3σ corresponds to $0.2\,\mathrm{mm}$.

2) Inclined rotor shaft

In a more realistic setting, the position of the two bearings that mount the rotor in the stator are uncertain (Fig. 1). The positions can again be expressed in polar coordinates (R_1, θ_1) and (R_2, θ_2) such that

$$R_1, R_2 \sim \mathcal{N}(0, \sigma^2) \quad \text{and} \quad \theta_1, \theta_2 \sim \mathcal{U}(0, \frac{\pi}{3}), \tag{2}$$

where the σ has the same interpretation as for SE.

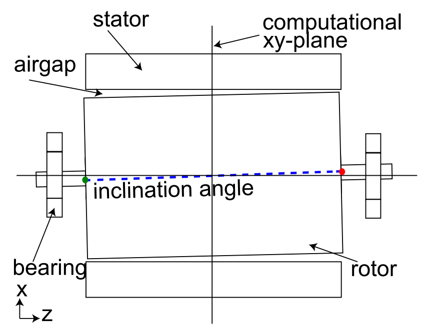

Fig. 1: Inclined rotor within stator. The green and the red dot correspond to the positions of the front and the back bearing, i.e. (R_1, θ_1) and (R_2, θ_2). The dashed line depicts the centre of the rotor throughout the stator bore.

B. Finite element method

The magnetostatic approximation of the Maxwell equations is sufficient to describe PMSMs, meaning that eddy currents and displacements currents can be neglected with respect to the source currents. Introducing the magnetic vector potential $\vec{A}$ one can retrieve from Ampère's law the elliptical PDE

$$\vec{\nabla} \times \left(\nu(\omega)\vec{\nabla} \times \vec{A}(\omega)\right) = \vec{J}_{\mathrm{src}} - \vec{\nabla} \times \vec{H}_{\mathrm{pm}}, \tag{3}$$

with Dirichlet boundary conditions. The dependency on the stochastic parameters is expressed by ω, ν is the reluctivity of the material, $\vec{J}_{\mathrm{src}}$ is the source current density and $\vec{H}_{\mathrm{pm}}$ the coercivity of the magnets. Applying the Galerkin approach will result in a system of equations, $\mathbf{K}_\nu(\omega)\mathbf{u}(\omega) = \mathbf{j}_{\mathrm{src}} + \mathbf{j}_{\mathrm{pm}}$, where $\mathbf{K}_\nu(\omega)$ is the FE system matrix e.g. [2], [3] and $\mathbf{u}(\omega)$ are the degrees of freedom. We use a 2D Ansatz with $\vec{A} = \sum_j u_j \vec{w}_j = \sum_j u_j \frac{N_j}{\ell_z} \vec{e}_z$, where $\vec{w}_j$ are dedicated edge shape functions related to the nodal finite elements $N_j(x, y, \omega)$ associated with a triangulation of the machine's cross-section. ℓ_z denotes the machine length and $\vec{e}_z$ is a unit vector in the z-direction. The EMF is calculated using the loading method [4]. The geometric variations are modelled without remeshing the finite element triangulation in order to reduce numerical noise in the stochastic outputs.

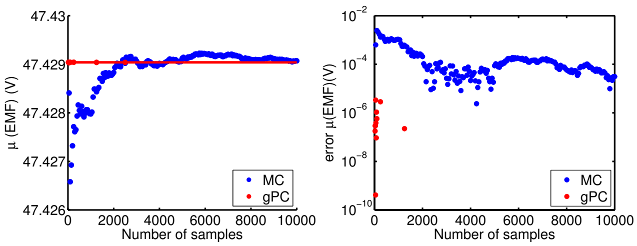

Fig. 2: Results for RSM-MC on a 17 x 5 tensor grid compared with the standard gPC approach for the expectation value of the EMF.

C. generalized Polynomial Chaos and response surface models

Besides the standard approach, MC, the multivariate gPC gains interest in UQ. The main advantage of gPC is the fast convergence for low dimensional problems [5]. Let $\vec{X}$ be a random vector of dimension d such that $\vec{X} = (X_1, \ldots, X_d)$ where X_i are independent random variables. Its response function $Y = Y(\vec{X})$ can then be approximated by the gPC expansion on a tensor grid

$$Y \approx \sum_{k=0}^{p} y_k \Phi_k(\vec{X}), \tag{4}$$

where y_k are the expansion coefficients and $\Phi_k(\vec{X})$ are orthogonal polynomial basis functions, i.e. $\mathbb{E}[\Phi_i(\vec{X})\Phi_j(\vec{X})] = \mathbb{E}[\Phi_i]\delta_{ij}$. The gPC bases for normal and uniform distributions are constructed from Hermite polynomials and Legendre polynomials, respectively [6].

A RSM is constructed by sampling the points of a $m \times n$ tensor grid, requiring mn FE simulations. In the subsequent UQ, the quantities of interest for intermediate parameter sets are obtained by cubic spline interpolation.

III. Results and Discussion

For SE, four simulation approaches are compared. The first one is a standard gPC approach on a 5x5 tensor grid requiring 25 FE evaluations. The second one is a standard MC approach with 1500 randomly chosen FE evaluation points. The third and fourth approach construct RSMs in the [R,θ] space with a 17x5 and 5x5 tensor grid, respectively. The 5x5 tensor grid corresponds to the one used in the standard gPC approach. The 17x5 tensor grid only requires 85 FE evaluations. While on the response surface grids a MC simulation with 10^8 samples is performed, using cubic spline interpolation for retrieving values for the EMF in between grid points with negligible numerical effort. Table I compares the numerical values for SE obtained by the four approaches. Fig. 2 shows the comparison between RSM-MC on a 17x5-tensor grid and the standard gPC. The standard MC approach did not yet converge after 1500 samples. For the RSM-MC approach proposed here the estimated interpolation error (Int. Err.) is bigger for the 5x5 grid than for the 17x5 grid, as has to be expected.

The effect of rotor inclination is studied by using the RSMs in combination with MC using 10^8 samples.

Table I: Numerical results of the EMF for SE.

Method	Exp. Value (V)	St. Dev. (V)	Int. Err.
stand. gPC (5x5 grid)	47.4290	$1.38883 \cdot 10^{-2}$	-
stand. MC (1500 samp.)	47.4286	$1.29809 \cdot 10^{-2}$	-
RSM-MC (5x5 grid)	47.4290	$1.38881 \cdot 10^{-2}$	10^{-6}
RSM-MC (17x5 grid)	47.4290	$1.38875 \cdot 10^{-2}$	10^{-8}

Table II: Numerical results of the EMF for the inclined shaft.

Method	Exp. Value (V)	St. Dev. (V)	Int. Err.
RSM-MC (5x5 grid)	47.4257	$6.94342 \cdot 10^{-3}$	10^{-6}
RSM-MC (17x5 grid)	47.4257	$6.94302 \cdot 10^{-3}$	10^{-8}

Each sample corresponds to the positions of the two bearings, which in turn correspond to two points in the RSMs. The intermediate slices correspond to points along a path in the RSM. The EMF of an inclined rotor configuration is found by interpolating and averaging along the path in the RSM. Table II shows the numerical results for the inclined rotor model. In comparison to SE, the standard deviations are much smaller. This confirms the fact that SE is the most pessimistic scenario, which is only attained in the inclined model when both bearings are equally displaced by chance.

IV. Summary

RSMs are constructed for the uncertain parameter space by FE evaluations for the points of a tensor grid. MC simulation on the basis of such RSMs outperforms the standard MC technique with FE simulation for the sample points and gPC on a comparable tensor grid. MC in combination with an RSM is particularly attractive for dealing with inclined rotors.

Acknowledgment

This work is supported by the German BMBF in the context of the SIMUROM project, by the 'Excellence Initiative' of the German Federal and State Governments and the Graduate School of CE at TU Darmstadt.

References

[1] B. M. Ebrahimi and J. Faiz, "Configuration impacts on eccentricity fault detection in permanent magnet synchronous motors", *IEEE Trans. Magn.*, vol. 48, no. 2, pp. 903–906, 2012.

[2] P. Monk, *Finite Element Methods for Maxwell's Equations*. Oxford: Oxford Univ. Press, 2003.

[3] S. J. Salon, *Finite Element Analysis of Electrical Machines*. Kluwer, 1995.

[4] M. A. Rahman and P. Zhou, "Determination of saturated parameters of PM motors using loading magnetic fields", *IEEE Trans. Magn.*, vol. 27, no. 5, pp. 3947–3950, 1991.

[5] L. Fagiano, M. Khammash, and C. Novara, "On the guaranteed accuracy of polynomial chaos expansions", in *Proc. 50th IEEE Conf. on Decision and Control – European Control Conf.*, IEEE, 2011, pp. 728–733.

[6] D. Xiu, *Numerical Methods for Stochastic Computations: A Spectral Method Approach*. Princeton Univ. Press, 2010.